Exhaustivity through the maxim of relation

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Abstract

I show that the *exhaustive* interpretation of answers can be explained as a conversational implicature through the *maxim of relation*, dealing with both the problematic *epistemic step* (Sauerland, 2004) and certain 'embedded' implicatures (Chierchia, Fox, & Spector, 2008). I assume a fairly standard maxim of relation, that captures the same intuition as Roberts' (2012) *contextual entailment*. I show that if a richer notion of meaning is adopted, in particular that of *attentive semantics* (Roelofsen, 2011), this maxim of relation automatically becomes strong enough to enable exhaustivity implicatures. The results suggest that pragmatic reasoning is sensitive not only to the information an utterance provides, but also to the possibilities it draws attention to.

Exhaustivity   In response to the question in (1), the utterance in (2), with neutral (i.e., final fall) intonation, is interpreted *exhaustively*: that John doesn't like the other colours.

(1)   Which colours (among red, blue, ...) does John like? $(p \lor q \lor (p \land q))$

(2)   He likes blue. (p) $\leadsto$ *he doesn't like red* $(\neg q)$

Exhaustivity has usually been considered a prime example of a Gricean (1975) *conversational implicature*. But how exactly this conversational implicature comes about has remained a topic of debate (e.g., Sauerland, 2004; Chierchia et al., 2008).

For simplicity in what follows, I assume the translations into propositional logic as given in parentheses in (1,2). These rely on, without loss of generality, (i) treating 'which' as an existential quantifier (since replacing (1) by 'there are colours that John likes' yields the same pattern), and (ii) assuming that blue and red are the only colours.

The main challenge   In order to explain exhaustivity as a conversational implicature, we must explain how it follows from the assumption that the speaker is *cooperative*. Now, it is relatively easy to derive from cooperativity the *quantity implicature* that the speaker in (2) does not have the belief that Mary came to the party (for otherwise the speaker should have said so). The main challenge has been to somehow strengthen this into the claim that the speaker believes that Mary didn't come, a strengthening, from $\neg \Box q$ to $\Box \neg q$, known as the *epistemic step* (Sauerland, 2004). All so-called 'Gricean' accounts of exhaustivity take the epistemic step by invoking some version of the assumption that the speaker is *maximally*

informed regarding the question under discussion, i.e., that for each alternative, the speaker knows either that it is true or that it is false. This derivation is given in (3), where just for brevity I use modal logic (any standard epistemic or doxastic interpretation suffices).

(3) $\quad \neg \Box q \qquad$ (quantity implicature)

$\qquad \dfrac{\Box q \vee \Box \neg q}{\Box \neg q} \therefore \quad$ (maximal informedness)

$\qquad\qquad\qquad\quad$ (exhaustivity)

However, the informedness assumption does *not* follow from the cooperativeness assumption (Chierchia et al., 2008). Hence, existing 'Gricean' theories all capture exhaustivity not as a case of conversational implicature, but as a case of *underspecification*. Like postulating lexical ambiguity, this is an unsatisfying last resort. But it is also inaccurate, because exhaustivity is implicated even in the explicit absense of a contextual informedness assumption, as can be seen by prefixing (1) with the disclaimer "I'm probably asking the wrong person, but...".

Solution The solution is based on the following intuition: the response (2) is *not sufficiently related* to the question (1), because it leaves the possibility that Mary will come *unattended*. In comparison, the equally informative response in (4) does not leave the Mary-possibility unattended, and does not implicate exhaustivity:

(4) John will come, or John and Mary / At least John will come. $\qquad (p \vee (p \wedge q))$[1]

The difference between (2) and (4) lies in the possibilities they draw attention to, as captured by *attentive semantics* (Roelofsen, 2011), in which meanings are sets of sets of worlds, i.e., sets of classical propositions. The meaning of a formula φ, $[\varphi]$, is conceived of as the set of possibilities it draws attention to, and its *informative content* $|\varphi|$ is given by $\bigcup[\varphi]$. For s a set of worlds, a proposition A *restricted to* s is $A_s = \{a \cap s \mid a \in A, a \cap s \neq \varnothing\}$.
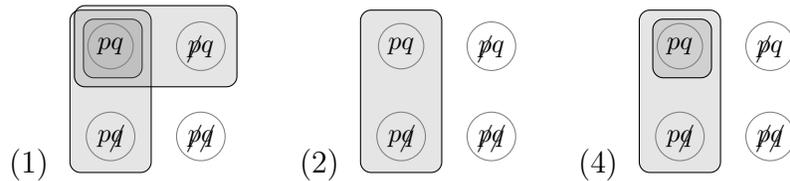
Definition 1 (Semantics of relevant fragment)
$[p] = \{\{w \mid w(p) = \textbf{true}\}\} \qquad [\neg \varphi] = \{\overline{|\varphi|}\}$
$[\varphi \wedge \psi] = ([\varphi] \cup [\psi])_{|\varphi| \cap |\psi|} \qquad [\varphi \vee \psi] = ([\varphi] \cup [\psi])_{|\varphi| \cup |\psi|} \ (= [\varphi] \cup [\psi])$

Entailment asks that the premiss is at least as informative *and attentive* as the conclusion:

Definition 2 (Entailment) $[\varphi]$ *entails* $[\psi]$, $[\varphi] \vDash [\psi]$, *iff* $|\varphi| \subseteq |\psi|$ *and* $[\psi]_{|\varphi|} \subseteq [\varphi]$

The example sentences have the following meanings (circles represent worlds):

To capture the intuition that (4), but not (2), is sufficiently related to the question, we adopt the following fairly standard maxim of relation:

Definition 3 (Maxim of Relation) *A speaker with information state s (set of worlds) should utter φ in response to ψ, only if $[\varphi]_s \vDash [\psi]$.*

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[1]This translation is in line with (Coppock & Brochhagen, 2013) on 'at least'.

This is a stripped-down version of Roberts (2012) *contextual entailment*, which requires entailment restricted to the *common ground*. Since the common ground contains the speaker's knowledge, my maxim logically follows from Roberts' (that is, if we would use the same semantics and entailment).[2]

The richer the semantics, the sparser entailment. In a classical semantics, (2) would entail (1), and no relation implicatures would be predicted. The same holds for, e.g., basic inquisitive semantics (Groenendijk & Roelofsen, 2009), as well as (Roberts, 2012). But with attentive semantics, rich enough to capture attentive content, this is no longer the case. For although (2) is more informative than (1), it is *less attentive*. On the other hand, (4) does entail (1), because it is both more informative and at least as attentive.

Therefore, for (2) to comply with the maxim of relation (i.e., to entail the question relative to the speaker's information), the speaker must know either:

1. that, if John goes, Mary goes too (because $[p]_{|\neg p \vee q|} = [p \wedge q] \vDash [p \vee q \vee (p \wedge q)]$); or
2. that, if John goes, Mary won't go (because $[p]_{|\neg p \vee \neg q|} = [p \wedge \neg q] \vDash [p \vee q \vee (p \wedge q)]$).

Of these, only the second is compatible with the quantity and quality implicatures, as shown in (5), and this yields exhaustivity (as before, using modal logic only for brevity):

(5)
$\Box p$ (quality implicature)
$\neg \Box q$ (quantity implicature)
$\underline{\Box(p \to q) \vee \Box(p \to \neg q)}$ (relation implicature)
$\Box \neg q$ (exhaustivity)

In sum, when attentive content is taken into account, the relation implicature of (2) enables the epistemic step.[3] In comparison, (4) complies with the maxim of relation as it is (because it entails (1)), hence it lacks a relation implicature, and no exhaustivity is implicated.

Discussion The epistemic step is overcome by adopting a semantics that is fine-grained enough to distinguish (2) from (4). Because only the former implicates exhaustivity, a theory of exhaustivity will need such a fine granularity anyway. As a consequence, entailment automatically becomes sparser, and the maxim of relation, as one finds it in the literature, becomes more powerful. In particular, it becomes sensitive to the possibilities that a response leaves unattended. This result generalizes to other cases of exhaustivity involving disjunction and existential quantification, and it suggests that (i) pragmatic reasoning is indeed sensitive to attentive content and (ii) exhaustivity is a conversational implicature. While I have used attentive semantics, the result generalizes to other (otherwise reasonable) semantics that lack the absorption laws, i.e., that distinguish (2) from (4) (e.g., Fine's (2013) *truth-maker semantics*, Ciardelli's (2009) *unrestricted inquisitive semantics*).

The epistemic step has been the main source of critique against 'Gricean' approaches to exhaustivity, but not the only one. The recent *localist-globalist* debate centers on exhaustivity implicatures that seem to arise from embedded positions (Chierchia et al., 2008):

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[2]We think Roberts' notion in fact conflates relatedness with a strong 'transparency' requirement that *all* dialogue participants know how an utterance is related to a question. A similar comparison could be drawn with Groenendijk and Stokhof's (1984) *pragmatic answerhood* (restricted to the *hearer*'s information state).

[3]It also shows that Roberts' requirement that all dialogue participants already *know* how an utterance is related, is too strong. In need only be required that the others *can figure out* how the utterance is related.

(6) Which books did every student read?
   - Every student read Othello or King Lear. $\rightsquigarrow$ *no student read both.*

The difficulty has been to derive the implicature 'no student read both', rather than the weaker 'not every student read both' that is typically generated by 'Gricean' approaches. Here, too, adopting a richer semantics provides a solution: because attentive content more closely resembles sub-sentential structure than informative content, it enables the maxims to see 'inside' sentences to some extent. Indeed, the theory outlined here already predicts the correct exhaustivity for the response in (6). Crucially, it improves over existing localist approaches in that the reasoning is still post-compositional/globalist.

Existing approaches to exhaustivity are typically formulated in terms of *alternatives*, treating exhaustivity as an answer to the question 'why did the speaker not give the more informative alternative, $p \wedge q$?'. The reason why these approaches fail to take the epistemic step is that mere ignorance is sufficient reason for not giving the more informative answer. In comparison, the present theory treats exhaustivity as an answer to the question 'why did the speaker not give the more *attentive* alternative, $p \vee (p \wedge q)$?'. Because ignorance is insufficient reason for omitting attentive content, something stronger is implicated: exhaustivity.[4]

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[4]While this rephrasing in terms of alternatives can be insightful for comparisons to existing approaches, it provides only an indirect view on the Gricean reasoning that is going on - the notion of 'more attentive alternative' does not occur in the maxims.